As a paradigmatic example of inverse boundary value problems, we focus on electrical impedance tomography (EIT), that is, on the determination of the conducting properties of an object by performing electrostatic measurements of current and voltage type on its boundary. The most important, and most famous, example is Calderón's, or inverse conductivity, problem. Other interesting cases in EIT are the inverse inclusion problem (a special case of the Calderón's one) and the inverse crack or cavity problem.

The main theoretical issues when facing an inverse problems are uniqueness, stability and reconstruction (by numerical methods). We show that, for the Calderón's problem, all these issues present severe challenges:

Uniqueness: fails in the anisotropic case.

Stability: these kinds of inverse boundary value problems are often ill-posed, that is, stability fails. For general conductivities we show that, as a consequence of non-uniqueness, instability may happen. To recover stability, suitable a priori assumptions on the unknown conductivities need to be imposed. Nevertheless, even with rather strong a priori assumptions, the stability is still very weak, namely, of logarithmic type (exponential or severe ill-posedness) as shown by Mandache’s argument. This has serious consequences when dealing with numerical methods.

Reconstruction: a naive least-square approach may fail in different respects. In fact, existence of a minimizer may fail, in the isotropic case. Even if a minimizer exists, it may be very different from the conductivity we are looking for, due to the instability of the problem.

Despite these difficulties, numerical methods can be devised to reconstruct the unknown conductivity by boundary measurements. When devising a numerical method, one crucial aspect is that to tackle the instability, for instance by applying a suitable regularization method. We focus on the regularization à la Tikhonov and we show that this a viable option for the inverse conductivity problem even in the challenging case of discontinuous conductivities.